Quantum mechanics. Department of physic. 7th semester.

Lesson №14. ***Quasi-classical approximation: of Bohr-Sommerfeld quantization rule.***

1. Time-independent Schrodinger equation in quasi-classical approximation

$$\psi = \exp\left(\frac{i}{\hbar}\sigma\right),$$

$$\frac{1}{2m}\left(\hbar\sigma'\right)^2 - \frac{i\hbar}{2m}\sigma'' = E - U(x),$$

$$\sigma = \sigma_0 + \left(\frac{\hbar}{i}\right)\sigma_1 + \left(\frac{\hbar}{i}\right)^2 \sigma_2 + \ldots$$

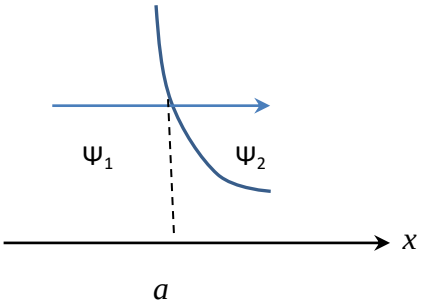
General solution of quasi-classical Schrodinger equation accurate up to σ_1 takes the form:

$\psi(x) = \frac{C_1}{\sqrt{p}}\exp\left(\frac{i}{\hbar}\int p(x)dx\right) + \frac{C_2}{\sqrt{p}}\exp\left(-\frac{i}{\hbar}\int p(x)dx\right)$ in the classically accessible region, and

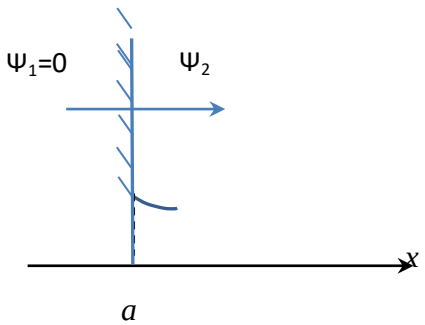
$\psi(x) = \frac{C_1}{\sqrt{|p|}}\exp\left(-\frac{1}{\hbar}\int |p(x)|dx\right) + \frac{C_2}{\sqrt{p}}\exp\left(\frac{1}{\hbar}\int |p(x)dx|\right)$ in the classically inaccessible region.

Usability condition is $\left|\lambda\frac{dp}{dx}\right| << |p|, \quad p(x) = \sqrt{2m(E-U(x))}; \quad \lambda = \frac{\hbar}{p(x)}.$

2. Boundary conditions for quasiclassical wave functions (the term of "cross-linking").

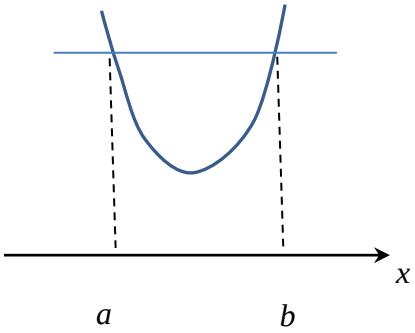


$$\frac{C}{2\sqrt{|p|}}\exp\left[\frac{1}{\hbar}\left|\int_x^a |p|dx\right|\right] \to \frac{C}{\sqrt{p}}\sin\left[\frac{1}{\hbar}\left|\int_a^x pdx\right| + \frac{\pi}{4}\right]$$



$$0 \to \frac{C}{\sqrt{p}}\sin\left[\frac{1}{\hbar}\left|\int_a^x pdx\right|\right]$$

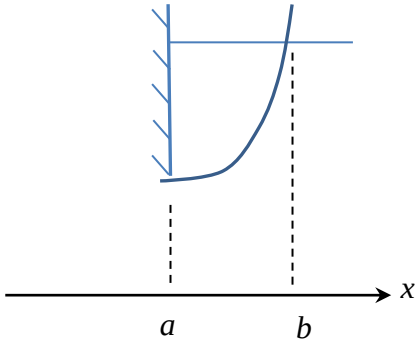
2.1. Bohr-Sommerfeld quantization rule for different boundary conditions



(smooth)

$$\int_a^b p dx = \pi\hbar\left(n+\frac{1}{2}\right)$$

or $$\oint p dx = 2\pi\hbar\left(n+\frac{1}{2}\right).$$



(vertical wall)

$$\int_a^b p dx = \pi\hbar\left(n+\frac{3}{4}\right).$$

or $$\oint p dx = 2\pi\hbar\left(n+\frac{3}{4}\right).$$

Task 1. Get a semiclassical expression for the energy levels of the linear oscillator. (НКК № 9.1)

Task 2. Get a semiclassical expression for the energy levels of the particle in an infinitely deep well $U(x)=\begin{cases}\infty, & x<0, x>a;\\ 0, & 0<x<a.\end{cases}$

Задача 3. Get a semiclassical expression for the energy levels of the particle in a field $U(x)=\begin{cases}\infty, & x<0;\\ mgx, & x>0.\end{cases}$ (НКК № 9.3)

Hometask: ЕК Гл.7 № 3-6.

LL – Landau L.D., Lifshitz E.M. Quantum Mechanics

НКК – Halitskii E.M., Karnakov B.M., Kohan V.I. Problems in Quantum Physics, 1981

ЕК – Elyutin P.V., Krivchenko V.D. Quantum Mechanics 1976